Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY

Winter Examination-2021

Subject Name: Problem Solving-I

Subject Code: 5SC02PRS1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 20/10/2021** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

- **a.** Define: Vector Subspace (01)
- **b.** Define: Direct sum of two subspaces (01)
- **c.** True/False: Any subset of linearly independent set is linearly independent set. (01)
- **d.** Find the two numbers whose sum is 4 and product is 8. (02)
- **e.** Express the number $\frac{(2+i)(1+2i)}{3+4i}$ into $x + iy$ form. (02)

Q-2 **Attempt all questions** **(14)**

- **a.** Check Whether the set R^+ of all positive real numbers with operations define as $x + y = x \cdot y$ and $kx = x^k$ is vector space. (07)
- **b.** Let S be the set of all elements of the form $(x + 2y, y, -x + 3y)$ in R^3, where $x, y \in R$. Show that S is a subspace of R^3. (04)
- **c.** Check whether the given set S is linearly dependent or linearly independent? (03)
$$S = \{e^x, xe^x, x^2e^x\}$$

OR

Q-2 **Attempt all questions** **(14)**

- **a.** Let W_1, W_2 and W_3 be subspaces of a vector space V such that $W_2 \subseteq W_1$, show that $W_1 \cap (W_2 + W_3) = W_2 + (W_1 \cap W_3)$. (05)
- **b.** Let $T: R^2 \to R^2$ be a linear transformation such that $T(1,2) = (2,3)$ and $T(0,1) = (1,4)$. Find formula for $T(x, y)$. (05)
- **c.** Find the rank of matrix by normal form, where $A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ (04)

Q-3		**Attempt all questions**	**(14)**
	a.	Solve: $\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0, \frac{dy}{dt}+5x+3y=0$	(07)
	b.	Solve: $(D^4+2D^2-3D)y=3\,e^{2x}+4\sin x$.	(05)
	c.	Find integrating factor of the given linear differential equation. $y\,dx+(x-y^3)dy=0$	(02)

OR

Q-3		**Attempt all questions**	**(14)**
	a.	If $f(z)=\begin{cases} u+iv & if\ z\neq 0 \\ 0 & if\ z=0 \end{cases}$ then C-R equation are satisfied at origin but $f^{'}(0)$ does not exists. Where $u(x,y)=\frac{x^3-y^3}{x^2+y^2}$ and $v(x,y)=\frac{x^3+y^3}{x^2+y^2}$	(07)
	b.	Find the Laurent series expansion of $f(z)=\frac{1}{(z+1)(z-3i)}$ about $z=-1$.	(07)

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Find order and degree of theequation $y=x\frac{dy}{dx}+\frac{x}{\frac{dy}{dx}}$.	(01)
	b.	Find argument of $cos\alpha+isin\alpha$	(01)
	c.	State Rank-Nullity theorem.	(01)
	d.	Find integrating factor for Non exact differential equation $x\,dy-y\,dx=0$.	(02)
	e.	Solve: $(D^2+4)y=0$	(02)
Q-5		**Attempt all questions**	**(14)**
	a.	If $f(z_0)=\oint_c \frac{3z^2+7z+1}{z-z_0}dz$, where c is the circle of radius 2 about origin, find the values of $f(1-i)$ and $f^{''}(1-i)$.	(07)
	b.	Show that $u(x,y)=e^{-2xy}\sin(x^2-y^2)$ is a harmonic function. Find the conjugate function $v(x,y)$ and an analytic function $f(z)$ for which $u(x,y)$is the real part.	(07)

OR

Q-5		**Attempt all questions**	
	a.	Solve the differential equation $y^{'''}-2y^{''}-21y^{'}-18y=3+4e^{-t}$ by using variation of parameters method.	(07)
	b.	Solve: $x^4\frac{d^3y}{dx^3}+2x^3\frac{d^2y}{dx^2}-x^2\frac{dy}{dx}+xy=1$	(07)
Q-6		**Attempt all questions**	**(14)**
	a.	Consider the basis $B=\{(1,-1,3),(0,1,-1),(0,3,-2)\}$ of $R^3(R)$ then find dual basis of B.	(05)
	b.	Find the 5^{th} roots of $(-\sqrt{3}+i)$.	(05)
	c.	Solve: $\frac{dy}{dx}+y\tan x=\sin 2x\,, y(0)=1$	(04)

OR

Q-6 **Attempt all Questions** **(14)**

a. Find a matrix P that diagonalize $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. Hence find A^{13}. (07)

b. Determine the nature, index and signature of the following quadratic forms (07)

$$Q(x_1, x_2, x_3) = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$$